Inference at $*$ 1 1 0
of proof for Lemma absval_eq:

1. $x$ : $\mathbb{Z}$
2. $y$ : $\mathbb{Z}$
$\vdash$ (if 0 $\leq$z $x$ then $x$ else -$x$ fi = if 0 $\leq$z $y$ then $y$ else -$y$ fi ) $\iff$ $x$ = $\pm$ $y$
by InteriorProof PERMUTE{1:n,
2:n,
3:n,
4:n,
5:n,
6:n,
7:n,
8:n,
9:n,
10:n,
11:n,
12:n,
13:n,
14:n,
15:n,
16:n,
17:n,
18:n,
19:n,
20:n,
18:n,
21:n,
19:n,
22:n,
23:n,
24:n,
25:n,
26:n,
24:n,
23:n,
27:n,
28:n,
29:n,
30:n,
31:n,
32:n,
33:n,
34:n,

35:n,
36:n,
37:n,
38:n,
39:n,
40:n,
41:n,
42:n,
43:n,
44:n,
45:n,
46:n,
47:n,
48:n,
46:n,
49:n,
47:n,
50:n,
51:n,
52:n,
53:n,
54:n,
52:n,
51:n,
55:n}

1: .....wf..... NILNIL

$\vdash 0 \leq\mathrm{z}\ x \in \mathbb{B}$

2: .....wf..... NILNIL

$\vdash \mathbb{B} \in \mathrm{Type}$

3: .....wf..... NILNIL

3. $0 \leq\mathrm{z}\ x = \mathrm{tt}$

$\vdash (0 \leq\mathrm{z}\ x = \mathrm{tt}) \in \mathbb{P}_1$

4: .....wf..... NILNIL

3. $0 \leq\mathrm{z}\ x = \mathrm{tt}$

$\vdash (\uparrow 0 \leq\mathrm{z}\ x) \in \mathbb{P}_1$

5: .....wf..... NILNIL

3. $0 \leq\mathrm{z}\ x = \mathrm{tt}$

$\vdash (0 \leq x) \in \mathbb{P}_1$

6: .....wf..... NILNIL

3. $0 \leq z\ x = tt$
$\vdash 0 \leq z\ x \in \mathbb{B}$
7: .....wf..... NILNIL

3. $0 \leq z\ x = tt$
$\vdash 0 \in \mathbb{Z}$
8: .....wf..... NILNIL

3. $0 \leq z\ x = tt$
$\vdash x \in \mathbb{Z}$
9: .....wf..... NILNIL

3. $0 \leq x$
$\vdash 0 \leq z\ y \in \mathbb{B}$
10: .....wf..... NILNIL

3. $0 \leq x$
$\vdash \mathbb{B} \in \text{Type}$
11: .....wf..... NILNIL

3. $0 \leq x$
4. $0 \leq z\ y = tt$
$\vdash (0 \leq z\ y = tt) \in \mathbb{P}_1$
12: .....wf..... NILNIL

3. $0 \leq x$
4. $0 \leq z\ y = tt$
$\vdash (\uparrow 0 \leq z\ y) \in \mathbb{P}_1$
13: .....wf..... NILNIL

3. $0 \leq x$
4. $0 \leq z\ y = tt$
$\vdash (0 \leq y) \in \mathbb{P}_1$
14: .....wf..... NILNIL

3. $0 \leq x$
4. $0 \leq z\ y = tt$
$\vdash 0 \leq z\ y \in \mathbb{B}$
15: .....wf..... NILNIL

3. $0 \leq x$
4. $0 \leq z\ y = tt$
$\vdash 0 \in \mathbb{Z}$
16: .....wf..... NILNIL

3. $0 \leq x$

4. $0 \leq$z $y =$ tt
$\vdash y \in \mathbb{Z}$

17:

3. $0 \leq x$
4. $0 \leq y$
$\vdash$ (if tt then $x$ else -$x$ fi = if tt then $y$ else -$y$ fi ) $\iff x = \pm\ y$

18: .....wf..... NILNIL

3. $0 \leq x$
4. $0 \leq$z $y =$ ff
$\vdash (0 \leq$z $y =$ ff$) \in \mathbb{P}_1$

19: .....wf..... NILNIL

3. $0 \leq x$
4. $0 \leq$z $y =$ ff
$\vdash (\uparrow y <$z $0) \in \mathbb{P}_1$

20: .....wf..... NILNIL

3. $0 \leq x$
4. $0 \leq$z $y =$ ff
$\vdash (y < 0) \in \mathbb{P}_1$

21: .....wf..... NILNIL

3. $0 \leq x$
4. $0 \leq$z $y =$ ff
$\vdash (\uparrow(\neg_b 0 \leq$z $y)) \in \mathbb{P}_1$

22: .....wf..... NILNIL

3. $0 \leq x$
4. $0 \leq$z $y =$ ff
$\vdash 0 \leq$z $y \in \mathbb{B}$

23: .....wf..... NILNIL

3. $0 \leq x$
4. $0 \leq$z $y =$ ff
$\vdash 0 \in \mathbb{Z}$

24: .....wf..... NILNIL

3. $0 \leq x$
4. $0 \leq$z $y =$ ff
$\vdash y \in \mathbb{Z}$

25: .....antecedent..... NILNIL

3. $0 \leq x$
4. $0 \leq$z $y =$ ff

$\vdash$ True

26: .....wf..... NILNIL

3. $0 \leq x$
4. $0 \leq$z $y =$ ff
5. $(\uparrow(\neg_b 0 \leq$z $y)) = (\uparrow y <$z $0)$

$\vdash \mathbb{P}_1 = \mathbb{P}_1$

27:

3. $0 \leq x$
4. $y < 0$

$\vdash$ (if tt then $x$ else -$x$ fi = if ff then $y$ else -$y$ fi ) $\Longleftrightarrow x = \pm\ y$

28: .....wf..... NILNIL

3. $0 \leq$z $x =$ ff

$\vdash (0 \leq$z $x =$ ff$) \in \mathbb{P}_1$

29: .....wf..... NILNIL

3. $0 \leq$z $x =$ ff

$\vdash (\uparrow x <$z $0) \in \mathbb{P}_1$

30: .....wf..... NILNIL

3. $0 \leq$z $x =$ ff

$\vdash (x < 0) \in \mathbb{P}_1$

31: .....wf..... NILNIL

3. $0 \leq$z $x =$ ff

$\vdash (\uparrow(\neg_b 0 \leq$z $x)) \in \mathbb{P}_1$

32: .....wf..... NILNIL

3. $0 \leq$z $x =$ ff

$\vdash 0 \leq$z $x \in \mathbb{B}$

33: .....wf..... NILNIL

3. $0 \leq$z $x =$ ff

$\vdash 0 \in \mathbb{Z}$

34: .....wf..... NILNIL

3. $0 \leq$z $x =$ ff

$\vdash x \in \mathbb{Z}$

35: .....antecedent..... NILNIL

3. $0 \leq$z $x =$ ff

$\vdash$ True

36: .....wf..... NILNIL

3. $0 \leq$z $x =$ ff
4. $(\uparrow(\neg_b 0 \leq$z $x)) = (\uparrow x <$z $0)$
$\vdash \mathbb{P}_1 = \mathbb{P}_1$

37: .....wf..... NILNIL

3. $x < 0$
$\vdash 0 \leq$z $y \in \mathbb{B}$

38: .....wf..... NILNIL

3. $x < 0$
$\vdash \mathbb{B} \in$ Type

39: .....wf..... NILNIL

3. $x < 0$
4. $0 \leq$z $y =$ tt
$\vdash (0 \leq$z $y =$ tt$) \in \mathbb{P}_1$

40: .....wf..... NILNIL

3. $x < 0$
4. $0 \leq$z $y =$ tt
$\vdash (\uparrow 0 \leq$z $y) \in \mathbb{P}_1$

41: .....wf..... NILNIL

3. $x < 0$
4. $0 \leq$z $y =$ tt
$\vdash (0 \leq y) \in \mathbb{P}_1$

42: .....wf..... NILNIL

3. $x < 0$
4. $0 \leq$z $y =$ tt
$\vdash 0 \leq$z $y \in \mathbb{B}$

43: .....wf..... NILNIL

3. $x < 0$
4. $0 \leq$z $y =$ tt
$\vdash 0 \in \mathbb{Z}$

44: .....wf..... NILNIL

3. $x < 0$
4. $0 \leq$z $y =$ tt
$\vdash y \in \mathbb{Z}$

45:

3. $x < 0$
4. $0 \leq y$
$\vdash$ (if ff then $x$ else -$x$ fi = if tt then $y$ else -$y$ fi ) $\iff x = \pm\ y$

46: .....wf..... NILNIL

3. $x < 0$
4. $0 \leq$z $y =$ ff
$\vdash (0 \leq$z $y =$ ff$) \in \mathbb{P}_1$

47: .....wf..... NILNIL

3. $x < 0$
4. $0 \leq$z $y =$ ff
$\vdash (\uparrow y <$z $0) \in \mathbb{P}_1$

48: .....wf..... NILNIL

3. $x < 0$
4. $0 \leq$z $y =$ ff
$\vdash (y < 0) \in \mathbb{P}_1$

49: .....wf..... NILNIL

3. $x < 0$
4. $0 \leq$z $y =$ ff
$\vdash (\uparrow(\neg_b 0 \leq$z $y)) \in \mathbb{P}_1$

50: .....wf..... NILNIL

3. $x < 0$
4. $0 \leq$z $y =$ ff
$\vdash 0 \leq$z $y \in \mathbb{B}$

51: .....wf..... NILNIL

3. $x < 0$
4. $0 \leq$z $y =$ ff
$\vdash 0 \in \mathbb{Z}$

52: .....wf..... NILNIL

3. $x < 0$
4. $0 \leq$z $y =$ ff
$\vdash y \in \mathbb{Z}$

53: .....antecedent..... NILNIL

3. $x < 0$
4. $0 \leq$z $y =$ ff
$\vdash$ True

54: .....wf..... NILNIL

3. $x < 0$
4. $0 \leq$z $y =$ ff
5. $(\uparrow(\neg_b 0 \leq$z $y)) = (\uparrow y <$z $0)$
$\vdash \mathbb{P}_1 = \mathbb{P}_1$

55:

3. $x < 0$
4. $y < 0$

$\vdash$ (if ff then $x$ else -$x$ fi = if ff then $y$ else -$y$ fi ) $\iff x = \pm\ y$

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